Anomalous far-infrared monochromatic transmission through a film of type-II superconductor in magnetic field

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Anomalous far-infrared monochromatic transmission through a lattice of Abrikosov vortices in a type-II superconducting film is found and reported. The transmitted frequency corresponds to the photonic mode localized by the defects of the Abrikosov lattice. These defects are formed by extra vortices placed out of the nodes of the ideal Abrikosov lattice. The extra vortices can be pinned by crystal lattice defects of a superconductor. The corresponding frequency is studied as a function of magnetic field and temperature in the framework of the Dirac-type two-band model. While our approach is valid for all type-II superconductors, the specific calculations have been performed for the $YBa_2Cu_3O_{7-\delta}$. The control of the transmitted frequency by varying the magnetic field and/or temperature is analyzed. It is suggested that the found anomalously transmitted localized mode can be utilized in the far-infrared monochromatic filters.

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I. INTRODUCTION

Infrared spectroscopy is one of the most important analytical techniques available to the modern science.[1] Due to intensive development of infrared spectroscopy in the recent decade, the construction of the different types of far-infrared monochromatic filters attracts a strong interest. Extraordinary optical transmission through nanostructures constructed as arrays of holes of a subwavelength diameter in metal films has been the subject of extensive study since detection of large enhancements in transmitted intensity was first reported.[2] Several mechanisms responsible for this enhancement have been discussed, including excitation of surface-plasmon polaritons of the film surfaces.[3,4] Such nanostructures can be used as monochromatic filters. However, it is impossible to control the transmitted resonant frequencies in such devices by an external field. In other words, these nanomaterials are not tunable. In this paper, we suggest an idea of a different type of a tunable far-infrared monochromatic filter consisting of extra vortices placed out of the nodes of the ideal Abrikosov lattice. These extra vortices are pinned by crystal defects in a type-II superconductor in strong magnetic field. The resonant transmitted frequencies can be controlled by two ways: changing the external magnetic field B and temperature T because the critical magnetic field B_{c2} depends parametrically on temperature.

Photonic crystals (artificial media with a spatially periodical dielectric function that were first discussed by Yablonovitch[5] and John[6]) are the subjects of growing interest due to various modern applications.[7,8] This periodicity can be achieved by embedding a periodic array of constituent elements ("particles") with dielectric constant ε_1 in a background medium characterized by dielectric constant ε_2. The first experimental evidence of a photonic-band structure was observed with metallic meshes in the terahertz range by Ulrich and Tacke.[9] Different materials have been used for the corresponding elements including dielectrics, semiconductors, and metals.[10–15]

Previous studies have investigated the photonic band-gap structure created by the propagation of light through a dielectric medium characterized by some dielectric constant with periodically located dielectric particles characterized by another dielectric constant.[10,11] The optical properties of low-dimensional metallic structures have also been investigated recently. For example, the optical transmission through a nanoslit array structure formed on a metal layer with tapered film thickness was analyzed in Refs. 12 and 13. The photonic-band structures of a square lattice array of metal or semiconductor cylinders, and of an array of metal or semiconductor spheres, were computed numerically in Ref. 14. However, a photonic crystal formed by placing superconducting (SC) particles in the nodes of the lattice has not been considered previously. Such a system is interesting, particularly, because of the unique optical properties of superconductors (see, for example, Refs. 16 and 17). In recent experiments SC metals (in particular, Nb) have been used as components in optical transmission nanomaterials. It was found that dielectric losses are substantially reduced in the SC metals relative to analogous structures made out of normal metals. Also, it should be mentioned that band edges tend to be sharper with the SC metals. The dielectric losses of such SC nanomaterial[18] were found to be reduced by a factor of 6 upon entering the SC state.

Photonic gaps are formed at frequencies ω, at which the dielectric contrast $\omega^2[\varepsilon_1(\omega)-\varepsilon_2(\omega)]$ is sufficiently large. Since the quantity $\omega^2\varepsilon(\omega)$ is included into the electromagnetic wave equation,[10,11] only metal-containing photonic crystals can maintain the necessary dielectric contrast at small frequencies due to their Drude-type behavior $\varepsilon_{\mathrm{metal}}(\omega)\sim -1/\omega^2$.[14,15] However, the damping of electromagnetic waves in metals can suppress many potentially useful properties of metallic photonic crystals.

A novel type of photonic crystal consisting of *superconducting* elements embedded in a dielectric medium was proposed in Ref. 19. Such photonic crystal provides the photonic band gap tuned by an external magnetic field and temperature. The photonic-band spectrum of the ideal triangular Abrikosov lattices in type-II superconductors studied as photonic crystals (ideal photonic crystal) has been calculated in Ref. 20.

In this paper, we calculate the photonic frequency spectrum of a photonic crystal with an extra vortex out of the node of the Abrikosov lattice (real photonic crystal) in a type-II superconductor in a magnetic field. The problem is solved in two steps: (1) we recall the procedure of the solution to the eigenvalue problem for the calculation of the photonic-band spectrum of the ideal Abrikosov lattice[19–21] and (2) we apply Kohn-Luttinger two-band model[22–25] to find the eigenfrequency spectrum of the Abrikosov lattice with one extra vortex inside. Based on the results of our calculations, we are suggesting a different type of a tunable far-infrared monochromatic filter consisting of extra vortices placed out of the nodes of the ideal Abrikosov lattice. These extra vortices are pinned by crystal defects in a type-II superconductor in strong magnetic field. As a result of the change of an external magnetic field B and temperature T, the resonant transmitted frequencies can be controlled. This paper is organized as follows. In Sec. II we analyze the mapping of the wave equation for the electromagnetic wave penetrating through the ideal Abrikosov lattice onto the Schrödinger equation for the wave function of an electron in the periodic field of a crystal lattice. In Sec. III we perform the calculations of the eigenfrequency corresponding to the electromagnetic wave localized due to the extra vortex out of the nodes of the ideal Abrikosov lattice applying Kohn-Luttinger two-band model, and frequency for the anomalous far-infrared monochromatic transmission is given. In Sec. IV we discuss our results proposing monochromatic filter based on type-II superconductor in the magnetic field. Conclusions follow in Sec. V.

II. ABRIKOSOV LATTICE AS AN IDEAL PHOTONIC CRYSTAL

Let us consider a system of Abrikosov vortices in a type-II superconductor that are arranged in a triangular lattice. We treat Abrikosov vortices in a superconductor as the parallel cylinders of the normal-metal phase in the superconducting medium. The axes of the vortices, which are directed along the $\hat{z}$ axis, are perpendicular to the surface of the superconductor. We assume the $\hat{x}$ and $\hat{y}$ axes to be parallel to the two real-space lattice vectors that characterize the two-dimensional (2D) triangular lattice of Abrikosov vortices in the film, and the angle between $\hat{x}$ and $\hat{y}$ is equal to $\pi/3$. The nodes of the 2D triangular lattice of Abrikosov vortices are assumed to be situated on the $\hat{x}$ and $\hat{y}$ axes.

For simplicity, we consider the superconductor in the London approximation,[17] i.e., assuming that the London penetration depth λ of the bulk superconductor is much greater than the coherence length ξ: $\lambda \gg \xi$. Here the London penetration depth is $\lambda=[m_e c^2/(4\pi n_e e^2)]^{1/2}$, where n_e is the electron density and m_e and e are the mass and the charge of the electron, respectively. The coherence length is defined as $\xi = c/(\omega_{p0}\sqrt{\epsilon})$, where $\omega_{p0}=2\pi c\omega_0$ is the plasma frequency and c is the speed of light. A schematic of Abrikosov lattices in type-II superconductors is shown in Fig. 1 (the presence of the pinned extra vortex is discussed in Sec. III). As it is seen from Fig. 1 the Abrikosov vortices of radius ξ arrange themselves into a 2D triangular lattice with lattice spacing $a(B,T)=2\xi(T)[\pi B_{c2}/(\sqrt{3}B)]^{1/2}$ (Ref. 20) at the fixed magnetic field B and temperature T. Here B_{c2} is the critical magnetic field for the superconductor. We assume the wave vector of the incident electromagnetic wave vector $\mathbf{k}_i$ to be perpendicular to the direction of the Abrikosov vortices and the transmitted wave can be detected by using the detector D.

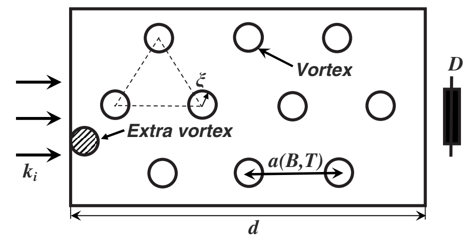

FIG. 1. Anomalous far-infrared monochromatic transmission through a film of type-II superconductor in the magnetic field parallel to the vortices. $a(B,T)$ is the equilateral triangular Abrikosov lattice spacing. ξ is the coherence length and the radius of the vortex. d denotes the length of the film. The shaded extra vortex placed near the boundary of the film and situated outside of the node of the lattice denotes the defect of the Abrikosov lattice.

Now let us follow the procedure used in Ref. 19 to obtain the wave equation for Abrikosov lattice treated as a two-component photonic crystal. In Ref. 19, a system consisting of superconducting cylinders in vacuum is studied. In contrast, the system under study in this paper consists of the cylindrical vortices in a superconductor, which is a complementary case (inverse structure) to what was treated in Ref. 19. For this system of the cylindrical vortices in the superconductor, we write the wave equation for the electric field $\mathbf{E}(x,y,t)$ parallel to the vortices in the form of a 2D partial differential equation. The corresponding wave equation for the electric field is

$$-\nabla^2\mathbf{E} = -\frac{1}{c^2}\epsilon\sum_{\{\mathbf{n}^{(l)}\}}\eta(\mathbf{r})\frac{\partial^2\mathbf{E}}{\partial t^2} - \frac{4\pi}{c^2}\frac{\partial \mathbf{J}(\mathbf{r})}{\partial t}, \tag{1}$$

where ϵ is a dielectric constant of the normal-metal component inside the vortices and $\eta(\mathbf{r})$ is the Heaviside step function, which is $\eta(\mathbf{r})=1$ inside of the vortices and otherwise $\eta(\mathbf{r})=0$. In Eq. (1) $\mathbf{n}^{(l)}$ is a vector of integers that gives the location of a scatterer l at $\mathbf{a}(\mathbf{n}^{(l)}) \equiv \sum_{i=1}^{d} n_i^{(l)}\mathbf{a}_i$ ($\mathbf{a}_i$ are real-space lattice vectors situated in the nodes of the 2D triangular lattice and d is the dimension of Abrikosov lattice).

At $(T_c-T)/T_c \ll 1$ and $\hbar\omega \ll \Delta \ll T_c$, where T_c is the critical temperature and Δ is the superconducting gap, a simple relation for the current density holds,[17]

$$\mathbf{J}(\mathbf{r}) = \left[-\frac{c}{4\pi\delta_L^2} + \frac{i\omega\sigma}{c}\right]\mathbf{A}(\mathbf{r}). \tag{2}$$

In Eq. (2) σ is the conductivity of the normal-metal component.

The important property determining the band structure of the photonic crystal is the dielectric constant. The dielectric constant, which depends on the frequency inside and outside of the vortex, is considered in the framework of the two-fluid model. For a normal-metal phase inside of the vortex, it is

$\epsilon_{\text{in}}(\omega)$ and for a superconducting phase outside of the vortex, it is $\epsilon_{\text{out}}(\omega)$ and can be described via a simple Drude model. Following Ref. 20 the dielectric constant can be written in the form

$$\epsilon_{\text{in}}(\omega) = \epsilon, \quad \epsilon_{\text{out}}(\omega) = \epsilon\left(1 - \frac{\omega_{p0}^2}{\omega^2}\right). \tag{3}$$

Equations (3) are obtained in Ref. 20 from a phenomenological two-component fluid model[26] by applying the following condition: $\omega_{\text{pn}} \ll \omega \ll \gamma$. Here ω_{pn} is the plasma frequency of normal conducting electrons and γ is the damping parameter in the normal conducting states.

Let us neglect a damping in the superconductor. After substituting Eq. (2) into Eq. (1), considering Eqs. (3) for the dielectric constant, and seeking a solution in the form with harmonic time variation of the electric field, i.e., $\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0(\mathbf{r})e^{i\omega t}$, where $\mathbf{E}_0 = i\omega\mathbf{A}/c$, we finally obtain the following equation:

$$-\nabla^2 E_z(x,y) = \frac{\omega^2\epsilon}{c^2}\left[1 - \frac{\omega_{p0}^2}{\omega^2} + \frac{\omega_{p0}^2}{\omega^2}\sum_{\{\mathbf{n}^{(l)}\}}\eta(\mathbf{r})\right]E_z(x,y), \tag{4}$$

where ω is the frequency and ω_{p0} is the plasma frequency. The summation in Eq. (4) goes over all lattice nodes characterizing the positions of the Abrikosov vortices. Equation (4) describes Abrikosov lattice as the two-component 2D photonic crystal. The first two terms within the bracket are associated with the superconducting medium, while the last term is related to vortices (normal-metal phase). Here and below the system described by Eq. (4) will be defined as an ideal photonic crystal. The ideal photonic crystal based on the Abrikosov lattice in type-II superconductor was studied in Refs. 20, 26, and 27. Equation (4) describing the Abrikosov lattice was solved in Ref. 20, where the photonic-band frequency spectrum $\omega = \omega(\mathbf{k})$ of the ideal photonic crystal of the vortices was calculated.

Equation (4) for the electric field can be mapped onto the 2D Schrödinger equation for an electron with effective mass m_0 in the periodic potential $W(\mathbf{r})$ of the 2D crystal lattice,

$$\left[-\frac{\hbar^2}{2m_0}\nabla^2 + W(\mathbf{r})\right]\psi_0(x,y) = \varepsilon_\omega\psi_0(x,y). \tag{5}$$

As a result of mapping in Eq. (5) the eigenfunction $\psi_0(x,y) = E_z(x,y)$; the periodic potential $W(\mathbf{r})$ is

$$W(\mathbf{r}) = -\hbar^2\epsilon\omega_{p0}^2/(2c^2m_0)\sum_{\{\mathbf{n}^{(l)}\}}\eta(\mathbf{r}) \tag{6}$$

and the eigenenergy is

$$\varepsilon_\omega = \hbar^2\epsilon[\omega^2 - \omega_{p0}^2](2m_0c^2)^{-1}. \tag{7}$$

It is important to note that according to Eq. (7) the eigenenergy of such electron ε_ω depends on the frequency ω.

Let us expand the periodic wave function $\psi_0(\mathbf{r})$ in terms of $u_{n0}(\mathbf{r})$, which are the periodic solutions of Eq. (5) corresponding to $k=0$,

$$\psi_0(\mathbf{r}) = \sum_{n\mathbf{k}} c_n(\mathbf{k})\exp[i\mathbf{kr}/\hbar]u_{n0}(\mathbf{r}), \tag{8}$$

where $c_n(\mathbf{k})$ are the coefficients of the expansion, which can be determined as a result of substituting Eq. (8) into Eq. (5), and n indicates the number of the band.

III. ABRIKOSOV LATTICE WITH AN EXTRA VORTEX AS A REAL PHOTONIC CRYSTAL

Let us consider an extra Abrikosov vortex pinned by some defect in the type-II superconducting material as shown in Fig. 1. This extra vortex contributes to the dielectric contrast by adding the term $\epsilon\omega_{p0}^2/c^2\eta(\xi - |\mathbf{r}-\mathbf{r}_0|)E_z(x,y)$, where $\mathbf{r}_0$ points out the position of the extra vortex, to the right-hand side of Eq. (4),

$$-\nabla^2 E_z(x,y) = \frac{\omega^2\epsilon}{c^2}\left[1 - \frac{\omega_{p0}^2}{\omega^2} + \frac{\omega_{p0}^2}{\omega^2}\sum_{\{\mathbf{n}^{(l)}\}}\eta(\mathbf{r}) + \frac{\omega_{p0}^2}{\omega^2}\eta(\xi - |\mathbf{r}-\mathbf{r}_0|)\right]E_z(x,y), \tag{9}$$

Equation (9) describes the type-II superconducting medium with the extra Abrikosov vortex pinned by a defect in the superconductor. We define the photonic crystal implying an extra Abrikosov vortex pinned by a defect as a real photonic crystal and it is described by Eq. (9).

Let us mention that the addition of the extra vortex pinned by some defect leads to a modification of the dielectric constant as it follows from Eq. (9). However, in our consideration the defect pinning the extra vortex does not take effect on the dielectric constant of the normal and superconducting components. Besides, let us emphasize that we consider no external current in the system.

After mapping Eq. (9) onto the Schrödinger equation for an electron with the effective electron mass m_0, we have

$$\left[-\frac{\hbar^2}{2m_0}\nabla^2 + W(\mathbf{r}) + V(\mathbf{r})\right]\psi(x,y) = \varepsilon_\omega\psi(x,y). \tag{10}$$

In Eq. (10) $\psi(x,y) = E_z(x,y)$ and the potential $V(\mathbf{r})$ is defined as

$$V(\mathbf{r}) = -V_0\eta(\xi - |\mathbf{r}-\mathbf{r}_0|), \quad V_0 = \hbar^2\epsilon\omega_{p0}^2/(2m_0c^2). \tag{11}$$

Equation (10) has the same form as Eq. (6) in Ref. 24. However, in our case the potential $V(\mathbf{r})$ is defined by Eq. (11) and corresponds to the potential of the impurity in the Schrödinger equation for an "electron" in the periodic field of the crystal lattice and in the presence of the "impurity."

Since the contribution to the dielectric contrast $\epsilon\omega_{p0}^2/c^2$ from an extra Abrikosov vortex has the same order of magnitude as the photonic band gap $\tilde{\Delta}$ of the ideal Abrikosov lattice calculated in Ref. 20, we expect the eigenfrequency level corresponding to the extra vortex to be situated inside the photonic band gap. Our calculations will demonstrate below that this expectation holds.

Let us apply to Eq. (10) the two-band model,[24] where two different neighboring photonic bands are described by wave functions $\varphi(\mathbf{r})$ and $\chi(\mathbf{r})$ and, therefore, introduce the two-component spinor as

$$\psi(\mathbf{r}) = \begin{bmatrix} \varphi(\mathbf{r}) \\ \chi(\mathbf{r}) \end{bmatrix}. \tag{12}$$

Note that Eq. (10) describes an electron in the periodic potential of the ideal crystal lattice $W(\mathbf{r})$ and in the potential of the impurity $V(\mathbf{r})$. If the solution corresponding to the absence of impurity $V(\mathbf{r})=0$ is known, the energy levels of the electron localized by the impurity can be obtained by replacing Eq. (10) by the Dirac-type equation according to Luttinger-Kohn model described in Refs. 22–24. This model implies a Dirac-type equation for the two-component spinor wave function. According to Ref. 24, the function $\varphi_n(\mathbf{r})$ defined as

$$\varphi_n(\mathbf{r}) = \sum_{\mathbf{k}} c_n(\mathbf{k})\exp[i\mathbf{k}\mathbf{r}/\hbar] \tag{13}$$

satisfied the set of the second-order partial differential equations. Considering only two neighboring bands corresponding to the wave function $\psi(\mathbf{r})$ given by Eq. (12) and described by wave functions $\varphi(\mathbf{r})$ and $\chi(\mathbf{r})$ in the limit $|\varepsilon_\omega^2 - \Delta_\omega^2|/(2\Delta_\omega^2) \ll 1$, this set of equations for $\varphi_n(\mathbf{r})$ can be reduced to the Dirac-type equations for the two-component spinor (12), which has the following form:[24]

$$[\varepsilon_\omega - \Delta_\omega - V(\mathbf{r})]\varphi(\mathbf{r}) + i\hbar s\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\chi(\mathbf{r}) = 0,$$

$$[\varepsilon_\omega + \Delta_\omega - V(\mathbf{r})]\chi(\mathbf{r}) + i\hbar s\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\varphi(\mathbf{r}) = 0. \tag{14}$$

In the system of Eqs. (14), as it follows from the mapping of the wave equation for the electric field in the Abrikosov lattice onto Eq. (10),

$$\Delta_\omega = \hbar^2\epsilon[\tilde{\Delta}^2 - \omega_{p0}^2]/(2m_0c^2) \tag{15}$$

is the forbidden band in the electron spectrum defined by Eq. (7) and $\boldsymbol{\sigma}$ are the Pauli matrices defined as

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \tag{16}$$

In Eqs. (14) $s=p/(\sqrt{3}m_0)$ has the dimension of the velocity and $\mathbf{p}$ is given by $(p_{cv\alpha})_\beta=\hbar k_0\delta_{\alpha\beta}$, where

$$k_0\delta_{\alpha\beta} = -i\left\{\left[\int u_{c0}^*(\mathbf{r})\,\boldsymbol{\nabla}\,u_{v0}(\mathbf{r})d^2r\right]_{cv\alpha}\right\}_\beta \tag{17}$$

and α,β can be x or y. The index c corresponds to the upper photonic band and the index v corresponds to the lower photonic band. Let us mention that the two-component model[24] described by the Dirac-type Eqs. (14) is necessary only for the "deep impurities" when the potential of the impurity has the same order of magnitude as the forbidden band: $|V_0/\Delta_\omega| \sim 1$. Note that in the limit, when the potential of the impurity would be much smaller than the gap, these Dirac-type equations would be reduced to the effective Schrödinger equation for the scalar wave function corresponding to the effective-mass approximation.

We have reduced the problem of the Schrödinger equation for a particle in the periodic potential $W(\mathbf{r})$ related to the system of the periodically placed vortices and an "impurity potential" $V(\mathbf{r})$ related to an extra vortex to a much more simple equation for the envelope wave functions. These wave functions imply the existence of two bands and contain only the potential of an impurity $V(\mathbf{r})$, while the periodic potential $W(\mathbf{r})$ enters only in the effective velocity s. Taking into account the two-band structure, the equation for two-component spinor wave function $\psi(\mathbf{r})$ has the form provided by Eq. (12). Note that Eq. (10) contains both the periodic function $W(\mathbf{r})$ corresponding to the ideal lattice and $V(\mathbf{r})$, describing the potential of an impurity. Without an impurity the energy spectrum would be described by two neighboring bands and the gap between them. Taking into account an impurity, we have reduced the problem to the approximation generalizing the effective-mass approximation and implying the two-band structure. Applying the standard two-band approach, we have obtained the effective Dirac-type Eqs. (14) for the envelope spinor wave function, which implies the periodicity provided by $W(\mathbf{r})$.

The condition for deep impurities is valid for an extra Abrikosov vortex only if $\omega_{\mathrm{up}}^2/\tilde{\Delta}_\omega^2 \sim 1$, which is true.[20] Note that $\omega_{\mathrm{up}}(x)$ and $\omega_{\mathrm{down}}(x)=\omega_{\mathrm{up}}(x)-\tilde{\Delta}(x)$ are the up and down boundaries of the photonic band gap, correspondingly.[20]

Defining the effective mass of a quasiparticle as

$$m_\omega = \Delta_\omega/s^2 = 3m_0^2\Delta_\omega/p^2, \tag{18}$$

and following the standard procedure of the quantum electrodynamics[28,29] to obtain from the system of Dirac-type Eqs. (14) the Klein-Gordon-type equation, finally we get:

$$[-\hbar^2s^2\nabla^2 + 2m_\omega s^2V(\mathbf{r})]\Psi(\mathbf{r}) = (\varepsilon_\omega^2 - m_\omega^2s^4)\Psi(\mathbf{r}). \tag{19}$$

This Klein-Gordon-type equation has the form of the 2D Schrödinger equation for a particle in the cylindrical potential well with the eigenvalue

$$\mathcal{E}_\omega = (\varepsilon_\omega^2 - m_\omega^2s^4)/(2m_\omega s^2). \tag{20}$$

The set of the eigenvalues $\mathcal{E}_\omega^{(nm)}$ and eigenfunctions $\Psi^{(nm)}$ corresponds to the quantum numbers $n=0,1,2,\ldots$ and $m=\ldots,-2,-1,0,1,2,\ldots$. Our particular interest is only discrete eigenstate corresponding to the localized eigenfunction, which is characterized by the lowest discrete eigenvalue $\mathcal{E}_\omega^{(00)}$. Equation (19) was solved in Ref. 30 and the solution for the discrete lowest eigenstate is

$$\mathcal{E}_\omega^{(00)} = -\frac{2\hbar^2}{m_\omega\xi^2}\exp\left(-\frac{2\hbar^2}{m_\omega\xi^2V_0}\right) \tag{21}$$

and

$$\Psi^{(00)}(\mathbf{r}) = \begin{cases} C_1, & |\mathbf{r}-\mathbf{r}_0| < \xi, \\ C_2\log[2\hbar(2m_\omega|\mathcal{E}_\omega^{(00)}|)^{-1/2}|\mathbf{r}-\mathbf{r}_0|^{-1}], & |\mathbf{r}-\mathbf{r}_0| > \xi, \end{cases} \tag{22}$$

where the constants C_1 and C_2 can be obtained from the condition of the continuity of the function $\Psi^{(00)}(r)$ and its derivative at the point $|\mathbf{r}-\mathbf{r}_0|=\xi$.

In terms of the initial quantities of the Abrikosov lattice, the eigenfrequency ω of the localized photonic state can be obtained by substituting Eqs. (15), (18), and (20) into Eq. (21). As the result, we finally obtain

$$\omega(x) = [\omega_{\mathrm{up}}^4(x) - A(x)]^{1/4}, \tag{23}$$

where $x = B/B_{c2}$ and the function $A(x)$ is given by

$$A(x) = \frac{16c^4 k_0^2(x)}{3\epsilon^2 \xi^2} \exp\left[- \frac{8k_0^2(x)c^4}{3\epsilon^2 \omega_{\mathrm{up}}^2(x)\xi^2 \omega_{p0}^2} \right] \tag{24}$$

and $k_0(x)$ in Eq. (24) can be obtained from Eq. (17) and is defined below through the electric field of the lower and higher photonic bands of the ideal Abrikosov lattice.

The electric field $E_z(x,y)$ corresponding to this localized photonic mode can be obtained by substituting the initial quantities of the Abrikosov lattice from Eqs. (15), (18), and (20) into Eq. (22), and we get

$$E_z^{(00)}(\mathbf{r}) = \begin{cases} \widetilde{C}_1, & |\mathbf{r} - \mathbf{r}_0| < \xi, \\ \widetilde{C}_2 B(x), & |\mathbf{r} - \mathbf{r}_0| > \xi, \end{cases} \tag{25}$$

where the constants $\widetilde{C}_1$ and $\widetilde{C}_2$ can be obtained from the condition of the continuity of the function $E_z^{(00)}(\mathbf{r})$ and its derivative at the point $|\mathbf{r}-\mathbf{r}_0| = \xi$, and

$$B(x) = \log[\xi(|\mathbf{r} - \mathbf{r}_0| \exp[-4k_0^2(x)c^4/\{3\epsilon^2[\widetilde{\Delta}^2(x) - \omega_{p0}^2]\xi^2\omega_{p0}^2\}])^{-1}]. \tag{26}$$

The function $k_0(x)$ is given as [see Eq. (17)]

$$k_0 \delta_{\alpha\beta} = -i \left\{ \left[\int E_{zc0}^*(\mathbf{r}) \nabla E_{zv0}(\mathbf{r}) d^2 r \right]_{cv\alpha} \right\}_{\beta}, \tag{27}$$

where $E_{zc0}(\mathbf{r})$ and $E_{zv0}(\mathbf{r})$ are defined by the electric field of the up and down photonic bands of the ideal Abrikosov lattice. The exact value of k_0 can be calculated by substituting the electric field $E_{zc0}(\mathbf{r})$ and $E_{zv0}(\mathbf{r})$ from Ref. 20. Applying the weak-coupling model[17] corresponding to the weak dielectric contrast between the vortices and the superconductive media $\omega_{p0}^2/\omega^2 |\Sigma_{\{\mathbf{n}^{(l)}\}} \eta(\mathbf{r}) - 1| \ll 1$, we use the approximate estimation of k_0 in our calculations as $k_0(x) \approx 2\pi/a(x) = \pi\xi^{-1}\sqrt{\sqrt{3}x/\pi}$.

Note that according to Eqs. (25) and (26), the maximum of the electric field corresponding to this localized mode is located at the center of the extra Abrikosov vortex pinned by a defect. This electric field $E_z^{(00)}(\mathbf{r})$ increases as the applied magnetic field B increases.

IV. RESULTS AND DISCUSSION

The approach we developed in Sec. III applies to the $YBa_2Cu_3O_{7-\delta}$ (YBCO) and we study the dependence of the photonic-band structure on the magnetic field. For the YBCO the characteristic critical magnetic field $B_{c2} = 5$ T at temperature $T = 85$ K was determined experimentally in Ref. 31. So we obtained the frequency corresponding to the localized wave for the YBCO in the magnetic-field range from $B = 0.72B_{c2} = 3.6$ T up to $B = 0.85B_{c2} = 4.25$ T at $T = 85$ K. Following Ref. 20, in our calculations we use the estimation $\epsilon = 10$ inside the vortices and for the YBCO $\omega_0/c = 77$ cm^{-1}. The dielectric contrast between the normal phase in the core of the Abrikosov vortex and the superconducting phase given by Eq. (3) is valid only for the frequencies below $\omega_{c1}: \omega < \omega_{c1}$, where $\omega_{c1} = 2\Delta_S/(2\pi\hbar)$, $\Delta_S = 1.76k_B T_c$ is the superconducting gap, k_B is the Boltzmann constant, and T_c is the critical temperature. For the YBCO we have $T_c = 90$ K and $\omega < \omega_{c1} = 6.601$ THz. It can be seen from Eqs. (23) and (25) that there is a photonic state localized on the extra Abrikosov vortex, since the discrete eigenfrequency corresponds to the electric field decreasing as a logarithm of the distance from an extra vortex. This logarithmical behavior of the electric field follows from the fact that it comes from the solution of 2D Dirac equation. The calculations of the eigenfrequency ω dependence on the ratio B/B_{c2}, where B_{c2} is the critical magnetic field, are presented in Fig. 2. According to Fig. 2, our expectation that the eigenfrequency level ω corresponding to the extra vortex is situated inside the photonic band gap is true. We calculated the frequency corresponding to the localized mode, which satisfies the condition of the validity of the dielectric contrast given by Eq. (3). According to Eqs. (25) and (26), the localized field is decreasing proportionally to $\log|\mathbf{r}-\mathbf{r}_0|^{-1}$ as the distance from an extra vortex increases. Therefore, in order to detect this localized mode, the length of the film d should not exceed approximately $10a(B/B_{c2})$, which corresponds to $d \lesssim 200$ μm for the range of magnetic fields for the YBCO presented in Fig. 2. For these magnetic fields $a \approx 20$ μm. Since the frequency corresponding to the localized mode is situated inside the photonic band gap, the extra vortex should be placed near the surface of the film as shown in Fig. 1. Otherwise, the electromagnetic wave cannot reach this extra vortex. Besides, we assume that this localized photonic state is situated outside the one-dimensional band of the surface states of two-dimensional photonic crystal. It should be mentioned that in a case of several extra

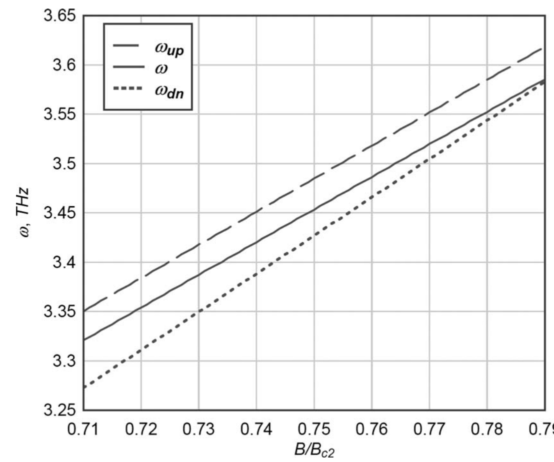

FIG. 2. The dependence of the photonic-band structure of the real Abrikosov lattice on B/B_{c2}. Solid line represents the eigenfrequency ω corresponding to the localized mode near the extra vortex in the real Abrikosov lattice given by Eq. (23). The dashed and dotted lines represent, respectively, the top ω_{up} and bottom ω_{down} boundaries of the photonic band gap of the ideal Abrikosov lattice according to Ref. 20.

vortices separated at the distance greater than the size of one vortex (it is the coherence length estimated for the YBCO by $\xi \approx 6.5\ \mu$m), the localized mode frequency for both vortices is also going to be determined by Eq. (23). The intensity of the localized mode in the latter case is going to be enhanced due to the superposition of the modes localized by the different vortices. In the case of far-separated extra vortices, we have neglected the vortex-vortex interaction. Thus, the existence of other pinned by crystal defects vortices increases the intensity of the transmitted mode and improves the possibility of this signal detection. Note that at the frequencies ω inside the photonic band gap ($\omega_{\rm down} < \omega < \omega_{\rm up}$), the transmittance and reflectance of electromagnetic waves would be close to 0 and 1, correspondingly, everywhere except the resonant frequency ω related to an extra vortex. The calculation of the transmittance and reflectance of electromagnetic waves at this resonant frequency ω is a very interesting problem, which will be analyzed elsewhere.

Let us mention that as B/B_{c2} increases, ω asymptotically converges to $\omega_{\rm down}$ up to the crossing point of $\omega_{\rm up}$ and $\omega_{\rm down}$ at $B/B_{c2} \approx 0.85$.[20] The reason for this convergence of ω and $\omega_{\rm up}$ with the increment of B/B_{c2} is that the symmetric potential always implies the discrete level corresponding to the eigenenergy of a particle in a 2D space.[32]

The extra Abrikosov vortex resulting in the defect in photonic crystal can be pinned by a crystal lattice defect, for example, dislocation.[33] It is shown that the presence of the extra vortex qualitatively influences the optical properties of the type-II superconductor in external magnetic field. Hence, it would be very useful to analyze the influence of different dislocation microstructures on the specific optical transmission in analyzed materials. The principles of a microdesign of twinning dislocation structures for superconductive properties improvement of the YBCO in magnetic fields were analyzed in Ref. 34. Magneto-optics in near fields as well as a neutron scattering[35] seem to be useful for detecting the extra vortex.

Above we presented the calculations for the YBCO. However, it should be mentioned that our approach can be applied to a wide variety of type-II superconducting materials, where Abrikosov lattice exists in the range of magnetic field $B_{c1} < B < B_{c2}$. While Fig. 2 implies the plasma frequency ω_{p0} corresponding to YBCO, the behavior shown in Fig. 2 is general and valid for any type-II superconducting films. Just in this case, we should replace the YBCO plasma frequency ω_{p0} by the plasma frequency corresponding to the other type-II superconductor.

V. CONCLUSIONS

We considered a type-II superconducting medium with an extra Abrikosov vortex pinned by a defect in a superconductor. By applying the mapping of the corresponding electromagnetic wave equation onto the two-band model, the Dirac-type equation was obtained. By solving this equation, we theoretically demonstrated the properties of such Abrikosov lattices as real photonic crystals. The discrete photonic eigenfrequency corresponding to the localized photonic mode is calculated as a function of the ratio B/B_{c2}, which parametrically depends on temperature. This photonic frequency increases as the ratio B/B_{c2} and temperature T increase. Moreover, since the localized field and the corresponding photonic eigenfrequency depend on the distance between the nearest Abrikosov vortices $a(B,T)$, the resonant properties of the system can be tuned by controlling the external magnetic field B and temperature T. Based on the results of our calculations, we can conclude that it is possible to obtain a different type of a tunable far-infrared monochromatic filter consisting of extra vortices placed out of the nodes of the ideal Abrikosov lattice, which can be considered as real photonic crystals. These extra vortices are pinned by crystal defects in a type-II superconductor in strong magnetic field. As a result of the change of an external magnetic field B and temperature T, the resonant transmitted frequencies can be controlled.

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